Now, if the function u be assumed of such value as to make the factor z in this equation vanish, we shall have the two equations,

$$\frac{du}{dx}+u\log x=0, \text{ and } u\frac{dz}{dx}=ax^{-x}.$$

From the former, we get by integration, $u=e^x x^{-x}$, and the value of u reduces the latter to $e^x\frac{dz}{dx}=a$ or $dz=ae^{-x}dx$. Whence $z=-ae^{-x}+C$.

Therefore $y=uz=e^x x^{-x}(-ae^{-x}+C)=x^{-x}(Ce^x-a)$, which is the required solution.

Also solved by *L. C WALKER*, *G. B. M. ZERR*, *F. P. MATZ*, and *J. SCHEFFER*.

131. Proposed by F P. MATZ. Sc. D.. Ph. D.. Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Integrate $2/x$, with regard to $d[\sqrt{(1-x^2)}]$.

Solution by the PROPOSER.

According to the conditions of the problem, we may write $d[F(x)]/d[\sqrt{(1-x^2)}]=2/x....(1)$.

$$\therefore F(x)=2\int\frac{d[\sqrt{(1-x^2)}]}{x}=-2\int\frac{dx}{\sqrt{(1-x^2)}}=\sec^{-1}\left[\frac{1}{2x^2-1}\right]$$

$=\cos^{-1}(2x^2-1)....(2)$, which is the integral required.

Also solved with various results by *H. C. WHITAKER*. *J. SCHEFFER*, and *G. B. M. ZERR*.

132. Proposed by JOHN M. COLAW, A. M., Monterey. Va.

What expression derived from the *polar* equation of a curve is equivalent to the expression for dy/dx derived from the *Cartesian* equation of the same curve? Prove work with $\rho=2r\cos\theta$.

Solution by LON C. WALKER. A.M.. Professor of Mathematics, Petaluma High School. Petaluma, Cal.

Let $x=\rho\cos\theta$, $y=\rho\sin\theta$, then

$$\frac{dy}{dx}=\frac{\sin\theta(d\rho/d\theta)+\rho\cos\theta}{\cos\theta(d\rho/d\theta)-\rho\sin\theta}....(1).$$

From $\rho=2r\cos\theta$, we get $d\rho/d\theta=-2r\sin\theta$. Substituting this value of $d\rho/d\theta$ in (1), and reducing, we have

$$\frac{dy}{dx}=\frac{r-x}{y}....(2).$$

Now substituting the value of x and y in $\rho=2r\cos\theta$, we get $x^2+y^2=2rx$; from which $dy/dx=(r-x)/y$, the same as in (2).

The angle the tangent makes with the radius vector is $\rho(d\theta/d\rho)$, which is often used in the same way as dy/dx; for instance, in trajectories, etc.

Also solved by *G. B. M. ZERR*, and *J. SCHEFFER*.

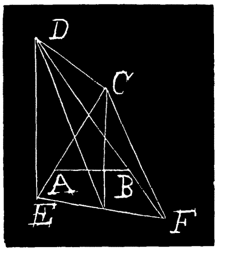
MECHANICS.

130. Proposed by W. J. GREENSTREET. M. A.. Editor of The Mathematical Gazette. Stroud. Gloucestershire, England.

Two particles are projected from A and B on the same level at α, β to horizon, and in vertical planes with which AB makes angles θ, φ. They meet and coalesce into a single particle. Find the height of the latus rectum of the snbsequent path above the level of A and B.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia. Pa.

Let D be the point of meeting of the two particles; mass of each, unity; C, the projection of D on the plane of AB; DE, DF, the tangents to the two paths, respectively; G, the mid-point of EF. Then DG is the direction of the resultant of DE and DF. Let $AB=a$, $\angle CAB=\theta$, $\angle CBA=\varphi$.



Then $AC=a\sin\theta/\sin(\theta+\varphi)$, $BC=a\sin\varphi/\sin(\theta+\varphi)$.

The equation to the path through AD is $m=n\tan\alpha-gn^2/2v^2\cos^2\alpha$, when $n=a\sin\theta/\sin(\theta+\varphi)$.

$$m=\frac{a\sin\theta\tan\alpha}{\sin(\theta+\varphi)}-\frac{ga^2\sin^2\theta}{2v^2\sin^2(\theta+\varphi)\cos^2\alpha}=h\ldots.(1).$$

The equation to the path through BD is $p=q\tan\beta-gq^2/2v_1^2\cos^2\beta$, when $q=a\sin\varphi/\sin(\theta+\varphi)$;

$$p=\frac{a\sin\varphi\tan\beta}{\sin(\theta+\varphi)}-\frac{ga^2\sin^2\varphi}{2v_1^2\sin^2(\theta+\varphi)\cos^2\beta}=h\ldots.(2).$$

From (1) and (2),

$$v_1^2=\frac{agv^2\sin^2\varphi\cos^2\alpha}{\cos^2\beta[ga\sin^2\varphi+2v^2\cos^2\alpha\sin(\theta+\varphi)(\sin\varphi\tan\beta-\sin\theta\tan\alpha)]}.$$

$$\tan DEC=dm/dn=\tan\alpha-gn/v^2\cos^2\alpha=\tan A.$$

$$\therefore \tan A=\tan\alpha-\frac{ag\sin\theta}{v^2\sin(\theta+\varphi)\cos^2\alpha};\ \tan DCF=\frac{dp}{dq}=\frac{\tan\beta-gq}{v_1^2\cos^2\beta}=\tan B.$$

$$\therefore \tan B=\tan\beta-\frac{ag\sin\varphi}{v_1^2\sin(\theta+\varphi)\cos^2\beta}.$$